The Metanarrative Nature and Transcendence of Formalist Mathematical Guidelines

Qin'an Wang
Hebei Polytechnic Institute

Abstract: In the basic propositions of the formalist school of mathematical philosophy, meta narratives, foundationalism, and absolutism constitute the ideological characteristics of modernity. The G ö del incompleteness theorem indicates that formal proof is insufficient for demonstrating mathematical truth, which constitutes the main reason for rejecting formalism. With the decline of foundational thinking, the holistic mathematical view of formalism has reached the end of its historical development.

Keywords: Fundamentalism; formalism; Metamathematics

In the late 19th and early 20th centuries, in order to solve the mathematical foundation crisis caused by a series of paradoxes such as set theory, many mathematicians and mathematical groups devoted themselves to establishing mathematical foundation reconstruction work to avoid the emergence of paradoxes and contradictions. One of the most notable schools of thought is formalism. Due to the fact that the members of this school were all famous mathematicians at the time, its influence in the field of mathematics was greater than that of other schools of thought. The fundamental goal of formalism is to establish a solid, timeless, and once and for all mathematical foundation. This idea holds an extremely important position in modern mathematical concepts. The G ö del incompleteness theorem declared the bankruptcy of the formalist mathematical program, and the meta narrative beyond the formalist program became an inevitable historical choice.

1.The basic propositions of formalism

The pioneer and representative figure of formalist mathematical thought is the famous mathematician Hilbert. Although Hilbert was not a thorough formalist, his advocacy for representing mathematics as a formalized system, using symbolic logic to represent its objects as statements with only logical structures and no actual content, led to a close connection between Hilbert and formalism. His basic ideas also constitute the core and dominant content of formalism.

Formalists advocate using formal axiomatization systems to integrate the entire classical mathematics. The formalization of a mathematical system is to describe it in formal language, which needs to meet several conditions such as symbol system, formation rules, and transformation rules. The axiomatization of a mathematical system is achieved by selecting a few undefined primitive concepts (basic concepts) and unconditionally recognized mutually restrictive provisions (axioms) as starting points, and through rigorous logical reasoning, making a mathematical system a deductive system. The idea of laying the foundation of mathematics through formalization and axiomatization constitutes the essence of the Hilbert Program. The basic idea of the Hilbert Program is to represent classical mathematics as a formal axiom system and then prove its compatibility. The specific objectives include:

(1) Prove that every branch of classical mathematics can be axiomatized.

(2) Prove that every such system is complete, that is, any propositions that can be expressed within the system can be determined within the system.

(3) Prove that every such system is compatible.

(4) Prove that the models corresponding to each such system are isomorphic.

(5) Find a method to determine the provability of any proposition within a finite number of steps.

2.Metanarrative ideas and their deconstruction in formalist mathematical views

Although some ideas of formalism have begun to transcend mathematical concepts in the sense of modernity, especially the metaphysical assumptions on ontology and the fundamental positions of meta theory (meta mathematics and meta logic) still constitute the foundation of formalist mathematical views. Therefore, it can be asserted that formalists' abandonment of modern mathematical concepts is not thorough. In his famous speech "On Infinity," Hilbert expressed the view that mathematical foundations have the highest authority in mathematical knowledge: "In a sense, mathematics has become an arbitration court, the highest court that adjudicates fundamental problems - a decision with concrete basis that everyone can agree on, and every statement can be controlled based on."

Hilbert and other mathematicians proposed the aforementioned metamathematical theory in order to lay a solid foundation in mathematics, with the aim of providing a rational foundation for mathematical proof, reasoning, methods, rules, etc. However, under the scrutiny of postmodern concepts, the fundamental stance of this meta theory has been repeatedly questioned. The famous French postmodernist Leotard proposed in his book "The Postmodern Situation": "We no longer believe in the existence of a privileged metadiscourse that can capture the truth of every primary discourse once and for all. The view that claims to have a metadiscourse status is untenable. The so-called metadiscourse is just one of all discourse." From Leotard's absorption of the results of the Godel's theorem in postmodern theory and its application to postmodern thought, Leotard's questioning of metadiscourse should also be inspired by the role changes of Hilbert's metamathematics in the evolution of mathematical foundations.

Although the basic tenets of foundationalism, as a form of epistemology, have been deconstructed in their modern significance as a whole, it should also be noted that programs such as "meta narratives" and "grand plans" have a promoting effect on practical mathematical research. The achievements of mathematical foundationalism, especially its local, technical, and methodological aspects, should be seen. For example, after canceling the requirement of determinism, Gen Cen proved the non contradiction of the arithmetic form system of natural numbers using the method of transfinite induction, demonstrating the importance of conceptual change for breakthroughs in the field of metamathematics.